

Dissipation-Scale Turbulence in the Solar Wind

G. G. Howes*, S. C. Cowley†, W. Dorland**, G. W. Hammett‡, E. Quataert*
and A. A. Schekochihin§

*Department of Astronomy, University of California, Berkeley, CA, USA.

†Department of Physics and Astronomy, UCLA, Los Angeles, CA, USA.

**Department of Physics and Center for Scientific Computing and Mathematical Modeling,
University of Maryland, College Park, MD, USA.

‡Princeton Plasma Physics Laboratory, Princeton, NJ, USA.

§King's College, Cambridge, UK and Department of Physics, Imperial College, London, UK.

Abstract. We present a cascade model for turbulence in weakly collisional plasmas that follows the nonlinear cascade of energy from the large scales of driving in the MHD regime to the small scales of the kinetic Alfvén wave regime where the turbulence is dissipated by kinetic processes. Steady-state solutions of the model for the slow solar wind yield three conclusions: (1) beyond the observed break in the magnetic energy spectrum, one expects an exponential cut-off; (2) the widely held interpretation that this dissipation range obeys power-law behavior is an artifact of instrumental sensitivity limitations; and, (3) over the range of parameters relevant to the solar wind, the observed variation of dissipation range spectral indices from -2 to -4 is naturally explained by the varying effectiveness of Landau damping, from an undamped prediction of $-7/3$ to a strongly damped index around -4 .

Keywords: Plasma turbulence—solar wind—kinetic damping—gyrokinetics

PACS: 52.35.Ra, 52.30.Gz, 96.25.Qr, 96.50.Tf

INTRODUCTION

One of the principal measurements in the study of solar wind turbulence is the magnetic field fluctuation frequency spectrum derived from *in situ* satellite measurements. At 1 AU, the one-dimensional energy spectrum in spacecraft-frame frequency typically shows, for low frequencies, a power law spectrum with slope of $-5/3$, suggestive of a Kolmogorov-like inertial range [1, 2]; a spectral break is typically observed at around 0.4 Hz, with a steeper power law at higher frequencies, often denoted the dissipation range in the literature, with a spectral index that varies from -2 to -4 [3, 4]. The general consensus is that the $-5/3$ portion of the spectrum is the inertial range of an MHD turbulent cascade, but the dynamics responsible for the break and steeper portion of the spectrum is not well understood. Various explanations for the location of the break in the spectrum have been proposed: that it is coincident with the proton cyclotron frequency in the plasma [5, 3, 6], or that the fluctuation length scale has reached either the proton Larmor radius [7, 8] or the proton inertial length [9, 10]. The steepening of the spectrum at higher wavenumbers has been attributed to proton cyclotron damping [1, 5, 3, 6], Landau damping of kinetic Alfvén waves [7, 8, 9], or the dispersive nature of whistler waves [11].

To unravel the underlying physical mechanisms at work in the solar wind requires an understanding of turbulence in weakly collisional, magnetized plasmas. Early theories of MHD turbulence proposed an isotropic cascade of turbulent energy [12, 13], but nu-

merical simulations [14] demonstrated an inherent anisotropy in the presence of a mean magnetic field. An evolving anisotropic theory [15, 14, 16, 17, 18] has emerged which rests upon two central hypotheses: the Kolmogorov hypothesis of locality in wavenumber space [19], and the conjecture that in strong turbulence the linear wave periods maintain a critical balance with the nonlinear turnover timescales. The anisotropic nature of the turbulence means the frequency for nonlinear energy transfer is dominated by the perpendicular wavenumber, $k_{\perp} v_{\perp}$, where \perp denotes the component perpendicular to the mean magnetic field. Assuming balanced turbulence with equal Elsässer energy fluxes in either direction along the mean field, the one-dimensional magnetic energy spectrum in the MHD regime scales as $E_B(k_{\perp}) \propto k_{\perp}^{-5/3}$ and critical balance implies a scale-dependent anisotropy with $k_{\parallel} \propto k_{\perp}^{2/3}$ [18]. In the regime of electron MHD (EMHD) [20], one obtains $E_B(k_{\perp}) \propto k_{\perp}^{-7/3}$ and $k_{\parallel} \propto k_{\perp}^{1/3}$ [21, 22].

Are observations of turbulence in the solar wind consistent with these theoretical predictions? The energy in turbulent fluctuations is observed to be anisotropic [23] with $k_{\perp} > k_{\parallel}$ in the slow solar wind at scales of $k_{\perp} \rho_i \sim 10^{-3}$ [24], where ρ_i is the proton Larmor radius; this appears consistent with the prediction of a scale dependent anisotropy leading to nearly perpendicular wavevectors $k_{\perp} \gg k_{\parallel}$ at small scales. The imbalance between anti-sunward and sunward Elsässer spectra can reach nearly two orders of magnitude in the fast wind, while the slow wind has a much smaller imbalance, from a factor of a few to approximate equality [25, 26]. Thus, we believe the aforementioned theory of MHD turbulence to be relevant to the dynamics in the slow solar wind.

Although the large scales at which the turbulence is driven may be adequately described by MHD, the turbulent fluctuations at the small-scale end of the inertial range often have parallel wavelengths smaller than the ion mean free path; therefore, a kinetic description of this weakly collisional plasma is required to capture the turbulent dynamics. The slow, fast, and entropy modes are damped in a warm, collisionless plasma [27]; the Alfvén wave cascade, however, is undamped until it reaches the ion Larmor radius, $k_{\perp} \rho_i \sim 1$ [28, 29]. For a sufficiently large inertial range, wavevectors at this scale become nearly perpendicular with $k_{\perp} \gg k_{\parallel}$; thus, frequencies remain low compared to the ion cyclotron frequency $\omega < \Omega_i$, the nonlinear cascade to yet smaller scales is composed of kinetic Alfvén waves, and Landau damping by the ions and electrons can effectively dissipate the turbulence. The dynamics in this regime optimally described by a low-frequency limit of kinetic theory called gyrokinetics [30, 29]. Here we present a model aimed at following the nonlinear cascade of magnetic energy from fluid to kinetic scales while accounting for the kinetic dissipation of the turbulence.

ANALYTICAL MODEL

Consider a homogeneous magnetized plasma with a mean magnetic field of magnitude B_0 that is stirred isotropically at an outer scale wavenumber k_0 with velocity v_0 . We write the magnetic field fluctuations in velocity units, $b_k \equiv \delta B_{\perp}(k_{\perp}) / \sqrt{4\pi n_i m_i}$. The frequency of nonlinear energy transfer for Alfvénic fluctuations at a given perpendicular wavenumber is estimated to be $\omega_{nl} \sim k_{\perp} v_k = k_{\perp} b_k$. Assuming the locality of nonlinear interactions in wavenumber space and a constant energy cascade rate ε , the one-dimensional mag-

netic energy spectrum in the regime $k_{\perp}\rho_i \ll 1$ is given by

$$E_B(k_{\perp}) = \frac{b_k^2}{k_{\perp}} = C_{1m}\epsilon^{2/3}k_{\perp}^{-5/3}, \quad (1)$$

where C_{1m} is a dimensionless constant of order unity. The frequency of nonlinear energy transfer is

$$\omega_{nl} = C_{2m}\epsilon^{1/3}k_{\perp}^{2/3}, \quad (2)$$

where C_{2m} is another order unity constant; the parallel wavenumber can be determined by applying the critical balance conjecture, setting the linear Alfvén wave frequency equal to the nonlinear frequency $\omega = \omega_{nl}$.

In the kinetic Alfvén wave regime $k_{\perp}\rho_i \gg 1$, the dynamics are governed by the equations of Electron Reduced MHD [29], with characteristic fluctuations $v_k = \pm b_k k_{\perp} \rho_i / \sqrt{\beta_i + 2/(1 + T_e/T_i)}$. Applying the same procedure for this regime yields the one-dimensional magnetic energy spectrum

$$E_B(k_{\perp}) = \frac{b_k^2}{k_{\perp}} = C_{1k}\epsilon^{2/3} \frac{[\beta_i + 2/(1 + T_e/T_i)]^{1/3}}{\rho_i^{2/3}} k_{\perp}^{-7/3}, \quad (3)$$

and the nonlinear frequency

$$\omega_{nl} = C_{2k}\epsilon^{1/3} \frac{\rho_i^{2/3}}{[\beta_i + 2/(1 + T_e/T_i)]^{1/3}} k_{\perp}^{4/3}. \quad (4)$$

A continuity equation for the magnetic energy per unit mass at each wavenumber b_k^2 can be written as [31]

$$\frac{\partial b_k^2}{\partial t} = -\frac{\partial \epsilon(k_{\perp})}{\partial \ln k_{\perp}} + S(k_{\perp}) - 2 \frac{\gamma(k_{\perp})}{\omega(k_{\perp})} \omega_{nl}(k_{\perp}) b_k^2, \quad (5)$$

where the three terms on the right-hand side are the energy flux through wavenumber space, a source term, and a damping term. The energy cascade rate is modeled by

$$\epsilon(k_{\perp}) = k_{\perp} b_k^3 \left[C_{1m}^{-3} + \frac{C_{1k}^{-3} (k_{\perp} \rho_i)^2}{\beta_i + 2/(1 + T_e/T_i)} \right]^{1/2} \quad (6)$$

and the nonlinear frequency by

$$\omega_{nl}(k_{\perp}) = k_{\perp} b_k \left[\frac{C_{2m}^2}{C_{1m}} + \frac{C_{2k}^2}{C_{1k}} \frac{(k_{\perp} \rho_i)^2}{\beta_i + 2/(1 + T_e/T_i)} \right]^{1/2}. \quad (7)$$

In the damping term, γ/ω is determined from the linear gyrokinetic dispersion relation [30]. The order unity constants are taken to be $C_{1m} = C_{1k} = 2.5$ and $C_{2m} = C_{2k} = 2.2$ based on numerical simulation, as in Quataert and Gruzinov [32].

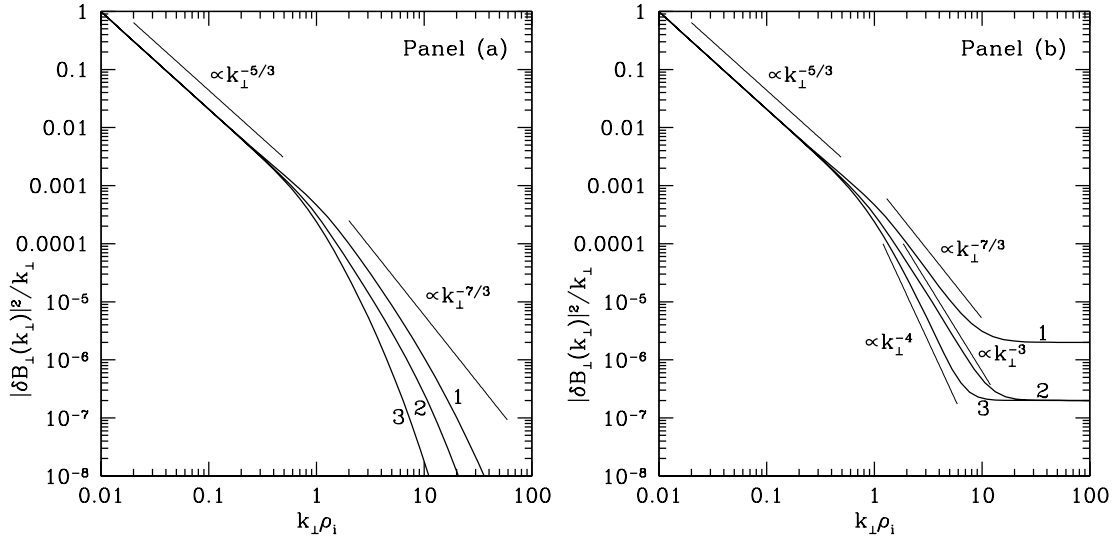


FIGURE 1. One-dimensional magnetic energy spectra for three gyrokinetic models: (1) $\beta_i = 0.5$, $T_i/T_e = 3$, (2) $\beta_i = 3$, $T_i/T_e = 0.6$, (3) $\beta_i = 0.03$, $T_i/T_e = 0.175$. All models use $k_0\rho_i = 3 \times 10^{-5}$. Panel (a) shows that all three spectra demonstrate a dissipative roll-off with a variation of spectral indices in the range $k_\perp\rho_i > 1$. Panel (b) adds a constant magnetometer sensitivity limit to each spectrum, yielding dissipation range spectra that more closely resemble power laws with a range of slopes from $-7/3$ to -4 .

RESULTS

The model given by equation (5) is solved numerically to obtain a steady state magnetic energy spectrum for a given set of the parameters ion plasma beta β_i , ion to electron temperature ratio T_i/T_e , and isotropic driving scale $k_0\rho_i$. Panel (a) of Figure 1 presents the solutions for three cases chosen to sample the observed parameter range in the solar wind [3, 33, 34]: (1) $\beta_i = 0.5$, $T_i/T_e = 3$; (2) $\beta_i = 3$, $T_i/T_e = 0.6$; and (3) $\beta_i = 0.03$, $T_i/T_e = 0.175$. All models use $k_0\rho_i = 3 \times 10^{-5}$. In the absence of dissipation, analytical theory predicts spectral indices of $-5/3$ in the MHD regime and $-7/3$ in the kinetic Alfvén wave regime; with the damping rate artificially set to zero, this model indeed recovers these results (not shown). Damping at $k_\perp\rho_i \gtrsim 1$ is sufficient to cause each spectrum in panel (a) to fall off more steeply than the undamped prediction of $-7/3$. The steady-state spectra obtained here clearly demonstrate the exponential roll-off characteristic of dissipation [35].

Observations of the magnetic fluctuation spectra at higher wavenumbers than the spectral break are widely interpreted to behave like a power law rather than an exponential decay. We suggest here that the power-law appearance of the spectrum in this range is an effect of limited magnetometer sensitivity; this sensitivity limit can be clearly seen in Figure 6 of Leamon et al. [3] at the high wavenumber end of the spectrum. The noise floor of a fluxgate magnetometer at frequencies $f > 1$ Hz is constant in units of $\text{nT}/\sqrt{\text{Hz}}$ [36], so we mock up the instrumental noise by specifying a constant background value of the one-dimensional energy spectrum. We choose the noise floor to be approximately two to three orders of magnitude below the spectrum value at the break. Panel (b) of

Figure 1 adds a constant sensitivity level at two (spectrum 1) or three (spectra 2 and 3) orders of magnitude below the spectrum value at $k_{\perp}\rho_i = 1$. The behavior of each spectrum in panel (b) in the range $k_{\perp}\rho_i > 1$ more closely resembles a power law than the exponential roll-off in the noiseless spectra; the steady-state solutions are well-fit by power laws with spectral indices $-7/3$, -3 , and -4 . In summary, the instrumental sensitivity limit is crucial in interpreting measured magnetic fluctuation spectra, and may produce spectra that appear to obey a power-law scaling even though the underlying spectrum is actually rolling off exponentially.

The spectral index in the dissipation range is observed to vary from -2 to -4 [3, 4]. Figure 1 shows that, over the range of the plasma parameters β_i and T_i/T_e measured in the solar wind, this variation can naturally be explained by the varying effectiveness of the damping of kinetic Alfvén waves via the Landau resonance. If Landau damping is negligible, the spectral index is expected to give a value of $-7/3$, close to the observed upper limit; over the range of parameters relevant to the solar wind, this cascade model gives a lower limit to the spectral index of about -4 , for example spectrum 3, consistent with observations. Hence, the varying effectiveness of Landau damping is sufficient to explain the observed variation of spectral indices, with the break occurring at the ion Larmor radius.

CONCLUSION

The physical mechanisms responsible for the spectral break and steeper dissipation range of the magnetic energy spectrum observed in the solar wind have not been conclusively identified. This paper presents a turbulent cascade model constructed to follow the magnetic fluctuation energy from the large scales in the MHD regime down to the small scales in the kinetic Alfvén wave regime, accounting for dissipation by kinetic processes. Due to the inherent anisotropy of MHD turbulence, the turbulence remains low frequency $\omega \ll \Omega_i$ and is optimally described by gyrokinetics. This picture of balanced, low-frequency turbulence is relevant to the slow solar wind.

The nonlinear cascade model given by (5) using the gyrokinetic damping rates is solved numerically to find steady-state solutions as presented in Figure 1. The cascade model predicts that, for wavenumbers above the break in the magnetic fluctuation energy spectrum, the spectrum undergoes a slow exponential cut-off. We argue that the widespread interpretation that this dissipation range shows power-law behavior is an artifact of limited magnetometer sensitivity. Over the range of parameters β_i and T_i/T_e measured in the solar wind, the varying strength of Landau damping naturally reproduces the observed variation of dissipation range spectral indices from $-7/3$ to -4 , with the spectral break occurring at the scale of the ion Larmor radius.

This model assumes that linear damping rates are relevant for turbulent fluctuations that are nonlinearly cascaded to smaller scales on the timescale of one wave period. Nonlinear gyrokinetic simulations of the turbulent cascade in the transition to the kinetic Alfvén wave regime are necessary to judge the validity of this assumption. This cascade model can be used as a tool to connect nonlinear numerical simulations to observations of turbulence in the solar wind. Further work to examine the importance of the proton cyclotron resonance in dissipation of solar wind turbulence is underway [34].

ACKNOWLEDGMENTS

Thanks to S. Bale for helping to apply this work to the solar wind. This work was supported by the DOE Center for Multiscale Plasma Dynamics, Fusion Science Center Cooperative Agreement ER54785 and the hospitality of the Aspen Center for Physics.

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